Criteria for Occurrence of LENR

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— Abstract —

There is a criterion for production of net energy in hot fusion experiments. It specifies the conditions that have to be created in a high temperature plasma for the output fusion power to exceed the power required to maintain the plasma. That criterion leads to the question of whether or not criteria can be developed for the occurrence of LENR. Having criteria that must be satisfied experimentally in order for LENR to occur might be useful for the design of LENR scientific experiments and the development of prototypes for LENR generators. There is considerable empirical evidence for the importance of both high deuteron concentrations and significant deuteron fluxes for production of LENR in electrochemical experiments. They are, respectively, the statics and the dynamics needed for the occurrence of LENR. Hence, we examine both of these factors in developing two quantitative criteria for when it is possible to produce LENR. Both of the needed conditions must be simultaneously satisfied in space and time. That requirement might partially explain why it is commonly challenging to produce LENR. Importantly, there are some experiments without clear fluxes of deuterons. So, it is necessary to consider alternative ways to get the dynamics needed for LENR to happen. Lattice vibrations, which are enhanced by increasing the temperature of an LENR experiment, are the most likely source of the deuteron motions. However, there does not seem to be a clear criterion for the required temperatures. The relative importance of fluxes and thermal vibrations in providing the dynamics needed for LENR remains to be determined.

1. Introduction

Hot and cold fusion share some physics and terminology, but are very different.[1] Hot fusion involves the production of plasmas with temperatures in the range of about 100 million degrees. The ions of light elements in such plasmas have velocities high enough to overcome their mutual electrostatic repulsion, so that they can make the nuclear contact (wavefunction overlap) needed for production of nuclear reactions and energy. Cold fusion involves the interaction of isotopes of hydrogen with each other and other nuclei on or in materials at relatively low temperatures and corresponding low energies. Hence, the most-used term for what was initially called cold fusion is now Low Energy Nuclear Reactions (LENR).

Even though there are such substantial differences, hot fusion and LENR both seek to generate controlled nuclear reactions for the production of useful energy. So, it is worth examining the much older field of hot fusion for items that might be useful to the study of LENR. One aspect of hot fusion that quickly attracts interest is a criterion for the production of net energy from hot fusion. It is called the Lawson Criterion, and is summarized in Section 2. That raises a question if there are also criteria for the production of LENR. The question is: *when* do LENR occur, that is, what conditions are needed for production of energy by LENR? Examination of that question is the motivation for this paper.

The third section provides some necessary background on the status of the scientific understanding and commercial exploitation of LENR. It includes a summary of information on *where* and *how* LENR occur. Both of those factors must be considered prior to discussing the quantitative requirements for when LENR occur. It will be argued that there are two primary requirements for when LENR occur. One is adequate concentrations of the reactants, which is basically the statics of LENR. However, the concentrations of reactants say nothing about their motions. Having high enough concentrations, while necessary, is insufficient. It is also necessary for the reactants to move and interact. So, the other requirement is enough motions of the reactants, the dynamics of the problem of producing LENR.

Sections 4 and 5 examine the respective requirements for having both sufficient concentrations and adequate dynamics at the same places in space and time to produce LENR. It is noted that the familiar challenge for producing LENR during experiments might have a simple cause. It is the frequent inability to achieve the dual requirement simultaneously at locations within a material such as palladium. Apparently, this double requirement for both high enough concentrations and adequate motions in some regions on or in materials is difficult to achieve simultaneously.

Section 6 deals with the quantitative LENR requirements that embody the criteria for concentrations and motions. Section 7 concludes with a discussion of the stated criteria for when LENR occur. The inevitable low sensitivities of calorimeters contribute to the problem of detecting LENR. So, alternative means to detect the occurrence of LENR are discussed. Section 7 also includes consideration of the needed additional work to more fully understand not only LENR, but also the criteria that quantify the requirements for production of LENR. It might turn out that such criteria prove to be useful for LENR, as is the Lawson Criterion for hot fusion.

2. The Lawson Criterion

Hot fusion research and development has a history that started in the middle of the last century, when the U.S. and USSR declassified some of their work on plasma physics. It was demonstrated in 1952 that nuclear fusion could release remarkable amounts of energy for very short times in a "hydrogen bomb."[2] A clear goal from that time was the more controlled and longer duration release of energy from fusion in a controllable and useful energy generator. That goal has proven very difficult to achieve. Seven decades of very costly global research has yet to result in a fusion system that can produce more energy than is required to run it.

In 1955, John Lawson was an engineer working at Harwell Laboratory in the UK. He wrote a then-classified report that quantified what was needed for practical production of nuclear fusion energy.[3] The report was declassified two years later, and a paper based on it was published.[4] Lawson reportedly stated that his main motivation for the work was that as an engineer he "felt the responsibility to 'pin down' the unrealistic expectations" of "his enthusiastic physics fellows."[5]

Lawson's key insight was based on a fundamental feature of plasmas—the fact that it takes a steady input power to maintain their temperatures and densities. Plasmas continuously lose energy to their surroundings due to particles and photons leaving them. Plasmas can be produced and maintained by the input of power electrically or optically, or by injection of energetic neutral atoms, if the plasma is adequately large. Fusion plasmas have two additional factors in the balance between input and output powers. Nuclear reactions lead to energetic charged products, such as helium nuclei (alpha particles), that can stay in the plasma and provide additional heating. And, neutrons from fusion reactions leave the plasma, since they have no electromagnetic interactions. Lawson basically stated that a necessary, but not sufficient, condition for net energy production by a fusion plasma was the need for fusion power production to exceed the power of the losses from the plasma. He was dealing with low-density magnetically confined plasmas. His criterion applies with some modifications to high-density inertially confined plasmas produced by lasers or electrical discharges, as will be noted below.

Plasmas in general, and fusion plasmas specifically, can be described by a number of parameters. They include their composition, that is, what elements and isotopes a plasma contains. The cross sections and reaction rates for a plasma consisting of only equal parts of deuterium and tritium are relatively high, so it is commonly assumed that a fusion plasma will be a 50-50 mixture of those two isotopes of hydrogen. The temperatures, densities and lifetimes of plasmas are critical factors for the Lawson Criterion, as will be detailed in this section. There are also many other descriptors for plasmas, but they do not figure in the Lawson Criterion, so we do not mention them. Books on plasma physics have the many details.[6]

The reasons for the importance of temperatures, densities and lifetimes in energy-producing plasmas are clear. The temperature is a measure of the energy of the electrons and ions in a plasma. Hence, it determines their speed, and high-speed ion collisions are needed to overcome the mutual repulsion of positive ions prior to fusion reactions. The ions have a distribution of speeds, and the cross sections for fusion vary with energy (speed). So, it is necessary to integrate over the distribution of ion speeds and the cross sections to obtain overall fusion reaction rates.

The density of deuterons and tritons is also critical to the rates of nuclear reactions. Plasmas with low densities simply do not have enough collisions and reactions for net power production. It is important to know that magnetically confined fusion energy plasmas have densities of only about one-millionth of the density of air on the earth's surface.[7] Their dilute character enables the ready escape of particles and photons.

The plasma lifetime is not as clearly an important parameter. It is a measure of how fast a plasma cools by loss of particles and photons. The energy density (energy/volume) of a plasma can be computed from its temperature and density. That energy density divided by the rate of energy loss (energy/volume x time) gives a lifetime for a plasma, which is called the confinement time. The input power to a plasma has to exceed the loss rate in order to maintain the plasma. The loss rate depends on the quality of the magnetic confinement of the fusion plasma.

Before considering more details about the Lawson Criterion, all of the means of energy input and loss for a plasma should be summarized. We already noted that plasmas can be heated by electrical currents, by photons or by injection of energetic neutral atoms. If fusion reactions occur and some of the particles they produce are retained in the plasma, they also contribute to the heating. The kinetic energy from helium nuclei is a prime example because they are charged and, hence, confined. Plasma losses come from ions and electrons, or photons from collisions in the plasma, which escape to the surroundings. And, if a plasma is producing fusions, neutrons will also carry off energy.

A plasma reaches "breakeven" when the energy produced by fusion is equal to the energy needed to produce and maintain the plasma. "Ignition" of a plasma occurs when the energy generated by fusion reactions is high enough to maintain the temperature of the plasma. Then, fusion can continue without external power input. That situation is much like how wood, once ignited, can continue to burn until the fuel is exhausted.

With this background, we can now sketch the derivation of the Lawson Criterion. Details are in a Wikipedia article on the topic.[8] Again, the issue is when the fusion power exceeds the power needed to maintain a plasma. The fusion power is computed first in the next paragraph. Then, the following paragraph considers the rate of energy loss from a fusion plasma. Those factors enable calculation of the Lawson Criterion, and then its extension to the "Triple Product" in the following paragraphs.

Fusion Power Input. The volume rate of fusion reactions (number per volume per time) is $n_D \times n_T \langle\sigma v\rangle = [n^2/4] \langle\sigma v\rangle$, where n_D and n_T are the densities of deuterons and tritons, each equal to half of the total plasma density n. The factor $\langle\sigma v\rangle$ is the average of the reaction cross section σ and velocity v over the distribution of velocities. The volumetric rate of energy production that goes into plasma heating is that reaction rate times the energy E of charged particles produced by fusion, which is E = 3.5 MeV for the D-T reactions.

Plasma Power Loss. The rate of energy loss from the plasma is computed by use of the confinement time τ, without having to consider separately the losses from the escape of particles and photons. As already noted, it is the energy density D (energy content per unit volume) divided by the power loss density P (rate of energy loss per unit volume), that is, D/P. For both the electrons and ions, $D = 3nkT$, where k is the Boltzmann Constant and T the temperature.

Lawson Criterion. Given the above, the Lawson Criterion for D-T fusion is embodied in the inequality: $[n^2/4] <\sigma v> E > 3nkT/\tau$. Rearranging, we have $n\tau > (12k/E)\ (T/<\sigma v>)$. The quantity $(T/<\sigma v>)$ can be calculated as a function of temperature, and is found to have a minimum numerical value. Substituting that value into the equation gives the minimum plasma requirement of $n\tau > 1.5 \times 10^{20}$ (sec/m^3) for D-T fusion for a temperature near 26 keV, which is close to 300 million degrees K.

The Triple Product. The Lawson Criterion was extended to what is called the "Triple Product" by multiplying both sides of the equation for the above inequality by the temperature T to get $nT\tau > (12k/E)\ (T^2/<\sigma v>)$. Again, the last factor can be computed and has a minimum value. Substitution then gives $nT\tau > 3 \times 10^{21}$ (keV x sec/m^3). The inequality can also be expressed with the temperature given in degrees K rather than keV. In that case, $nT\tau > 3 \times 10^{28}$ (deg K x sec/m^3). It is interesting that the inequality can be satisfied, and breakeven reached, with many combinations of n, T and τ. The three quantities are very different, but can effectively be "traded" for each other to achieve the minimum requirement.

This Triple Product has been widely used to assess the performance of many magnetic fusion devices over the decades. There are several plots of the Triple Product as a function of plasma temperature, with points that show the peak performance of various fusion experiments. Figure 1 shows one example on the left.[5] It is seen that most of the magnetic-confinement fusion experiments have been done with deuterium plasmas, since tritium is expensive and radioactive. The diagram on the left of Figure 1 also contains the approximate history of the increases in the Triple Product. Another plot, on the right in Figure 1, explicitly shows the Triple Product vs. year.[9] It clearly indicates that it has gotten more difficult to increase the Triple Product over the decades of magnetic confinement fusion research. The immense and expensive International Thermonuclear Experimental Reactor (ITER), now being built in the south of France, is aiming for a Triple Product of 5×10^{21} (keV x sec/m^3), as indicated in Figure 1. A discussion of the Triple Product for ITER is available.[10]

The Tokamak is the most widely studied hot fusion experimental design. Many other magnetically-confined plasma fusion experiments have been tried, and are still being developed.[9] A very different approach to fusion energy production is termed "inertial confinement." It uses extremely high-powered lasers[11] or immense electrical currents[12] to produce fusion plasmas of very high densities for very short times. The core idea, and the reason for the name, is to produce fusion energy before the very hot and dense plasma can overcome the inertia due to the mass of the ions and explode. The Lawson Criterion and Triple Product apply to inertially confined plasmas as well as to magnetically confined plasmas, as discussed above. A different form of the Triple Product is used for inertially confined plasmas.[8]

A natural question is whether or not it is possible to use the Lawson Criterion for LENR. That seems unlikely since the equations used for both the required power input to, and the inevitable power losses from, a hot plasma do not apply to solid materials. First, we consider the input side. The deuterons in a metallic material are remarkably mobile, since they do not have bound electrons. They have some similarities to the ions in a plasma due to that mobility and their being screened by mobile bonding electrons. However, there is no accepted means of relating the energy distributions of the deuterons to LENR reaction rates. Hence, there is no clear and accepted way to compute the LENR power from the experimental conditions. Second, we review the output side. The unavoidable power loss part of LENR is not simply estimated. The decay rates of LENR power for what is called "Heat After Death" have been measured.[13] However, they might depend on the materials geometry, and there is no theory for the LENR decay rate.

To summarize: On the input power side, the threshold for occurrence of LENR, and the variation in LENR rates above the threshold, are not known theoretically. On the output power side, it is not known theoretically how fast a material

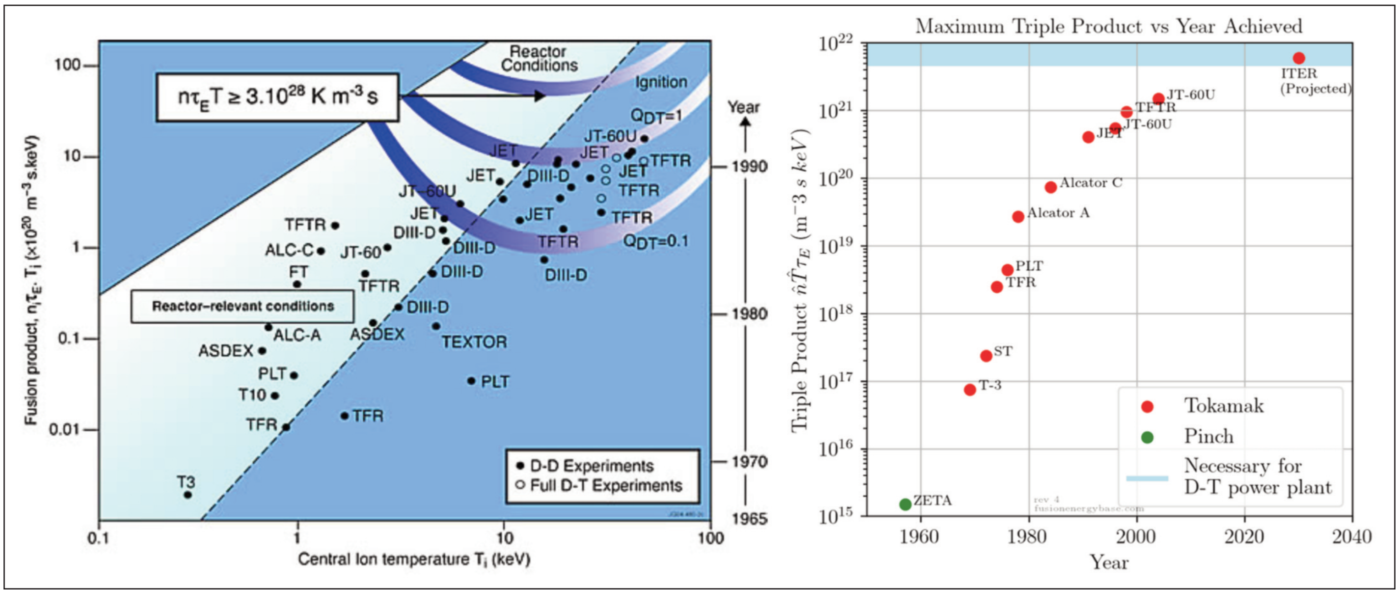

Figure 1. Left: Plot of the fusion Triple Product vs. the maximum ion temperature for various devices. Right: The Triple Product vs. years for various fusion plasma experiments (mostly Tokamaks).

in which LENR are occurring loses its ability to support continuing production of LENR. So, we do not have LENR analogs for either part of the hot fusion Lawson Criterion.

3. Where and How LENR Occur

If the Lawson Criterion is not applicable to LENR, but such a criterion is useful, is it possible to develop criteria for LENR? This paper is concerned with the conditions for which LENR can occur, that is, *when* is it possible to produce LENR. That issue, and criteria used to quantify it, are closely related to the *where* and *how* LENR occur. Those two factors are dealt with in another paper.[14] This section summarizes the main viewpoints and conclusions from that paper as a prelude to presenting and justifying in the following sections the criteria for when LENR occur.

Regarding the locations *where* LENR can occur on or in materials, there is ample evidence that reactions can happen on the surfaces of materials and in their interior. Both surfaces and the inside of crystalline materials are complex chemically and structurally. It appears that defects on and in materials play a role in the production of LENR. Surfaces of materials are commonly full of various defects with zero, one, two and three dimensions. There is a need for LENR experiments in which the type and density of surface effects are varied to see if they result in changes in LENR rates. Of the many defects of various dimensionality within materials, grain boundaries might be a favored location for LENR. It ought to be possible to measure the effect of varying the grain sizes, and with them the area of grain boundaries, in LENR materials. If it is found that small grain sizes and large grain areas favor the production of LENR, the science of LENR will be significantly clarified. Such a finding might also speed the commercialization of LENR. The relative importance of (a) surfaces, which are two-dimensional defects on the outside of lattices, and (b) grain boundaries, which are also two-dimensional defects within lattices, remains to be determined. That is a major question scientifically and practically.

Regarding *how* LENR occur, that is, the mechanism that causes LENR, there are many theories.[15] Only a few of them are adequately explored with (a) a clear concept, (b) equations based on the concept, and (c) numerical results from the equations for comparison with the results of past experiments, or the prediction of the results of future experiments. Of the several LENR theories that are more or less complete, that of Kálmán and Keszthelyi[16] can be used as a good example of (a) how LENR might occur and (b) how the theory can be tested. The work is thorough, and has (a) a clear and intelligible three-body concept, (b) second-order quantum mechanical equations based on the concept, (c) many numerical results from the equations for diverse nuclear reactions, and (d) significant successful comparisons with LENR data. The three bodies are the two reactants and a lattice nucleus that serves as a catalyst. The work shows both how LENR can occur with measurable rates, and how LENR occur without emission of significant energetic radiation. However, the work has yet to be adequately studied by scientists who are able to fully understand and derive the equations and also reproduce the calculations.

The Lawson Criterion, in both its basic form and as a Triple Product, is a single inequality, that is, one mathematical criterion. It is not clear if conditions for the occurrence of LENR can be expressed so simply. So, we next separately consider both the concentrations and dynamics of LENR reactants. This is also necessary, since one cannot computationally balance the input and output powers for LENR, as Lawson was able to do for hot fusion plasmas.

4. Concentrations of Reactants

The need for high concentrations of deuterons for occurrence of LENR has a long history. Early work by McKubre[17] and Kunimatsu,[18] and their colleagues, showed that a high "loading" of deuterons is needed. Loading is the ratio of the

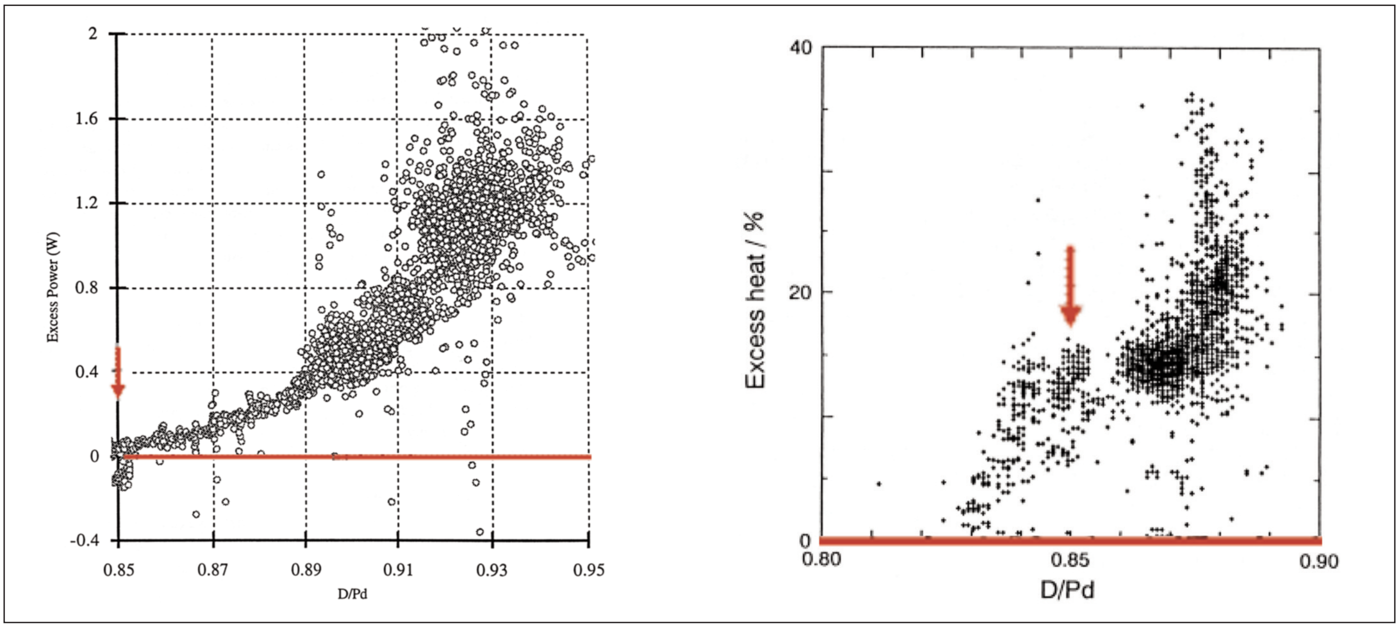

Figure 2. Left: Variation of excess power expressed in watts with the degree of loading X = D/Pd from 0.85 to 0.95 from McKubre. Right: Dependence of the excess power expressed as the percentage of the input electrical power from X = 0.80 to 0.90 from Kunimatsu. The arrows mark the position of X = 0.85, and the thick horizontal lines give the position of zero excess power.

number of deuterons in a metal, usually palladium, to the number of Pd atoms expressed as X = D/Pd. Data from 1992 are shown in Figure 2. Both data sets show that excess power increases approximately as the square of the degree of loading above a threshold in the range of X = 0.80 to 0.85. While the agreement is imperfect, the data do support each other, both evidently and because of their being obtained in distinct experiments in two very different settings, one in the U.S. and the other in Japan. Further, the quadratic dependence on the concentration of the reactants is familiar from the study of chemical reactions.[19] Many other results on the dependence of LENR heat production on the degree of loading have been given.[20] McKubre and Tanzella published data that show the much-trumpeted early failures to measure excess heat were due to the experimenters' inability to get to the necessary high degrees of loading.[21] The early results led to the widely-adopted view that high loading is a prerequisite for the occurrence of LENR. However, other work took issue with that view.

Storms found that he could produce LENR with average loading ratios as low as 0.01.[22] The theoretical work of Kálmán and Keszthelyi cited above indicates that LENR rates go as the product of the volumetric densities of the three bodies involved in their mechanism, namely (N_{Pd} x N_D x N_D) or $(N_D)^2$, where N_{Pd} is the density of catalytic Pd lattice nuclei and N_D is the deuterium density. This is the first explanation of the quadratic dependence of LENR on loading that is shown in Figure 2. Also, it indicates that the local, not overall average loading, is what is important. The high average loading achieved at great difficulty in the work of McKubre, Kunimatsu and others indicated high overall, and hence also high local loadings. However, the work by Storms and the Hungarian theoreticians indicates that it is only the *local* loading that is significant for occurrence of LENR. This might apply equally to the occurrence of LENR on surfaces or within materials. The issue of local and average loading raises a question about measuring the degree of local loading, that is, the loading as a function of position within a material. That does not appear to be possible at this time. X-ray diffraction can probe the interior of materials during experiments. However, it does not have the spatial resolution needed for determination of loading on and below a spatial scale of micrometers.

5. Dynamics of Reactants

Both chemical and nuclear reactions have two basic requirements: the availability of reactants and their interactions. We considered reactant availability by dealing with concentrations in the last section. It is a necessary, but not sufficient condition for LENR. Now, we consider the interactions of the reactants. We already noted that the occurrence of LENR requires both adequate loading and sufficient motions simultaneous in space and time. Getting enough reactants into place on or in a lattice is one challenge. Now, we can address the fact that the production of adequate motions to produce LENR is complex due to the presence of the lattice. As was the case for concentrations, the situation for fluxes of deuterons is based on measurements.

It is useful to review the similarities and differences between chemical reactions in liquids and in solids. When atoms or molecules are present in liquids, they move and interact quite freely due to the fluid nature of liquids and the thermal vibrations. However, atoms or molecules in solids are constrained by the lattices of the material in two ways—the directions in which they can move and the freedom of any motions. Lattice atoms both limit motions to particular directions and impede motions in those and other directions. These constraints on chemical reactions in solids also apply to nuclear reactions in solids at low energies. So, in the study of LENR, we have to contend with two competing situations. On one hand, whatever the mechanism for LENR, a lattice enables their occurrence. On the other hand, the lattice constrains the motions needed for interactions and low energy reactions.

The case for the necessity of deuteron motions is quite

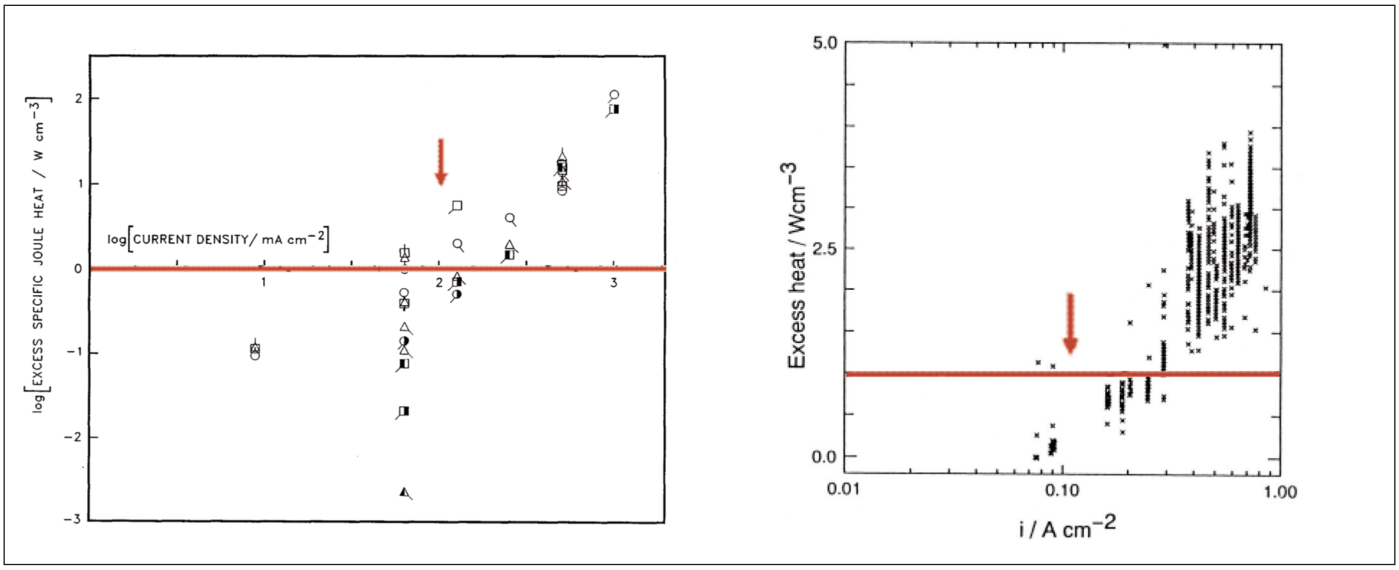

Figure 3. Excess heat in watts/cm^3 of the cathode. Left: Data from Fleischmann and Pons. Right: Data from Kunimatsu and his colleagues. The arrows give the positions of 100 mA/cm^2, and the thick horizontal lines give the position for 1 W/cm^3 of excess heat. The scatter in the Fleischmann-Pons data at low current densities might indicate that those results were near the limit of their ability to measure excess heat, or that other factors such as impurities influenced the results.

strong. McKubre and his colleagues produced an empirical equation for LENR power.[23] It contained two terms relevant to deuterium fluxes. One was the rate of change of loading, that is, the flow of deuterons into or out of a palladium cathode. The other was the electrolysis current density, which is related to fluxes, as will be discussed next. It was found early in the field that LENR production depends on the electrolysis current density, for which there is a threshold. Some data of that kind from the Fleischmann and Pons,[24] and from the Kunimatsu group,[18] are shown in Figure 3. In both cases, the threshold for the electrical current was near 100 mA/cm^2. Storms[25] made similar plots, and found threshold values from somewhat below 100 to about 200 mA/cm^2. The flux of ions associated with such electron fluxes will be considered in the next section.

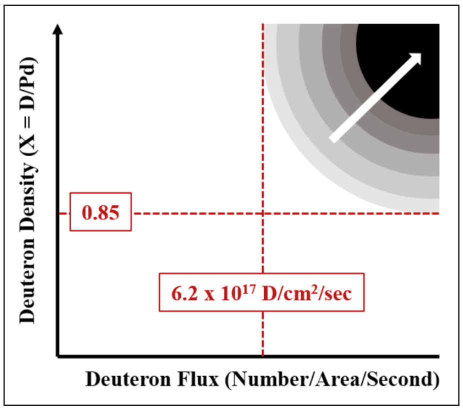

Figure 4. Plot of the concentrations and fluxes needed for LENR.

Besides the above indications of the importance of deuterium ion flux, there is other evidence for its relevance to producing LENR. Strong production of LENR in experiments at Energetics Technologies[26] and the Naval Research Laboratory[27] showed that changes in the input current (and power) of either sign can sometimes lead to strong increases in the output LENR power. Such changes can lead to increases in the flux of deuterons in the associated direction. The phenomenon of "Heat After Death," where LENR power continues to be produced after an electrochemical experiment is turned off, can be ascribed to the outflow of deuterons from a highly-loaded cathode. This possibility shows that deuteron fluxes out of a material might be as effective as fluxes into a material in producing LENR.

In addition to fluxes of deuterons, there is another possibility for deuteron motions in a lattice to cause LENR. That possibility is lattice vibrations of an extent adequate to cause deuterons to move from sites in the lattice into other nearby sites that contain a deuteron. We know that vibrations are what cause atomic motions in materials that lead to diffusion. We also know that high temperatures favor production of LENR. Is it only because they increase diffusion, and hence either or both the concentration of deuterons or their fluxes? Or, do the increased thermal vibrations at elevated temperatures increase interactions between nearby deuterons in regions that already have adequate concentrations of deuterons? A recent theoretical paper addresses the effects on LENR of lattice vibrations.[28] If both fluxes and vibrations can introduce the deuteron dynamics needed for interactions and nuclear reactions, which is dominant? And, how does the relative importance vary with temperature, loading and other parameters? These are among the basic scientific questions about the occurrence of LENR.

6. Quantitative LENR Requirements

Given the dual requirements of the concentrations and motions of reactants just discussed, we now consider the possibility that there are plots for those LENR requirements. Such plots might be somewhat similar to the graphics for hot fusion shown in Figure 1. One version of a plot of requirements for LENR is given in Figure 4. It shows that both adequately large concentrations and fluxes are needed for generation of LENR. One must get into the shaded region to produce LENR. Movement within the shaded region can give variable LENR rates. A threshold exists for both quantities, concentrations and rates. The numbers given for those thresholds will be considered in the following paragraphs. The diagram in Figure 4 can be viewed as a 2 x 2 matrix, where both the concentrations and the fluxes either are insufficient or adequate for LENR production. Only one of the four possible combinations is effective in causing LENR. There are many ways for experiments to fail to produce detectable LENR.

The threshold for the concentration C_T can be determined from the data plots in Figure 2. There it is seen that a concentration threshold of X = D/Pd = 0.85 suffices. In those plots, it was likely that the value was the average for the entire Pd sample. So, it should also suffice for local regions within a piece of palladium. The lattice constant of Pd is 3.86×10^{-8}, so the volume of a Pd unit cell = $(3.86 \times 10^{-8})^3 = 57.5 \times 10^{-24}$ cm^3. Hence, the concentration of palladium atoms is 1.7×10^{22} per cm^3. A loading of 0.85 corresponds to a deuteron density of about 1.5×10^{22} per cm^3.

The threshold for deuteron flux F_T is not quite so simple to determine. However, it is based on the measurements shown in Figure 3. It takes two electrons to electrolyze one water molecule, freeing two deuterons. As noted, several researchers found that the threshold for LENR is about 100 mA/cm^2. That is equivalent to $0.1 \times 6.24 \times 10^{18}$ electrons/sec $= 6.24 \times 10^{17}$ electrons/sec. Given the lattice constant of Pd of 3.86 A $= 3.86 \times 10^{-8}$ cm, the area of a unit cell of Pd = $(3.86 \times 10^{-8})^2 = 14.9 \times 10^{-16}$ cm^2. If we assume unity for the Faradaic efficiency, these values give (6.24×10^{17}) (ions/sec cm^2) $\times (14.9 \times 10^{-16})$ cm^2 / lattice area = 930 deuterons per lattice face per second. The flux of deuterons due to diffusion is the product of the diffusion coefficient and the concentration gradient of deuterons into the palladium. The actual flux of deuterons might be lower after a run starts, when the concentration gradient into the palladium is not as steep as it is at the outset of a run. Eventually, the palladium might be saturated with deuterons, so any further input flux has to be balanced by a flux of deuterons leaving the palladium or being consumed by LENR. Such an equilibrium, if established, could happen at different levels of average and local deuteron concentrations.

It is also possible to estimate the deuteron flux from how long it takes to load a palladium cathode rod of certain dimensions. Consider a Pd cathode 1 mm in diameter and 2 cm long, which loads to X = D/Pd = 0.6 in 1 hour. The volume is Area x Length $= \pi \times 10^{-2} (1/2)^2 = 0.78 \times 10^{-2}$ cm^3. The volume of a Pd unit cell = $(3.86 \times 10^{-8})^3 = 57.5 \times 10^{-24}$ cm^3. Hence, there are $(0.78 \times 10^{-2}) / (57.5 \times 10^{-24}) = 1.36 \times 10^{20}$ Pd cells, each with four Pd atoms. For the loading ratio of 0.6, there are 3.26×10^{20} deuterons in the Pd cathode. The cathode area is π 0.1 x 2 = 0.628 cm^2, hence there are $(0.628) / (14.9 \times 10^{-16}) = 4.2 \times 10^{14}$ lattice faces on the cathode. The deuteron entry rate is $(3.26 \times 10^{20}) / (4.2 \times 10^{14}) \times 3600 = 216$

ions per lattice area per second. This lower value results from the fact that it is averaged over the entire hour it takes to load the cathode to a value of X = 0.6.

With the values just discussed and shown in Figure 4, a single criterion for occurrence of LENR might be $C_T \times F_T > 0.85 \times 6.2 \times 10^{17} = 5.3 \times 10^{17}$ with units of D/cm^2/sec. The validity of this criterion depends on the sensitivity with which the occurrence of LENR can be determined. That topic is discussed in the next section. The ability to simply trade concentration for flux, and vice versa, within the shaded area of Figure 4, depends on the linearity of LENR rates with both factors. However, we already noted that LENR rates depend on the square of the deuteron concentration. Hence, the actual LENR rates in the shaded region of Figure 4 should have a complex dependence. Put another way, if Figure 4 had a third dimension giving LENR rates in the shaded region, the surface in that region would not be planar.

Whatever the variations within the shaded region, its boundary might be useful in guiding some electrochemical LENR experiments. The hyperbolic inequality in the last paragraph can be satisfied for various combinations of C_T and F_T. It is relatively easy to get to a loading value of X = 0.6 in electrochemical LENR experiments. That is well short of the value of 0.85 discussed above. If it is indeed possible to use an increased flux of deuterons to compensate for deficiencies in loading, it might appear that one could simply increase the current, at least in the early stages of a run, by the ratio of 0.85/0.6 to satisfy the inequality. However, the quadratic dependence of LENR rates on loading might prevent that simple approach. An even larger increase in current and flux might be needed to move an experiment across the boundary of the shaded region in Figure 4. High electron and ion currents are usually not used in the early stages of electrochemical LENR experiments. Also, the average, and probably also the local, concentrations of deuterons are usually low early in experiments. This discussion leads to the possibility of loading palladium cathodes to X = 0.6 in the initial stages of a run, and then significantly increasing the electron (and ion) currents. That might be done in the ratio of at least $(0.85/0.6)^2$ to compensate for the quadratic dependence of LENR on concentration. Whatever the needed increase in current, it ought to be possible to perform such experiments using the normal setups and instrumentation for electrochemical LENR experiments. One could vary the magnitude of the electrical current step upon achievement of a loading of X = 0.6, to see if LENR occur at some value of the higher current. The ratio of ion currents to electron currents as a function of loading is an open question.

There has been significant discussion of means of triggering LENR over the years.[29] A distinction has been made between loading materials with reactants, and then doing something else (called triggering) to initiate the production of LENR. An extreme example of separating loading and triggering was the experiment by McKubre and Tanzella.[30] They loaded wires of palladium and nickel electrochemically, and then sealed in the protons or deuterons by adding mercury to the electrolyte. That enabled them to move the wires to a calorimeter, where they were subjected to strong electrical pulses and showed evidence of LENR. The approach to loading, and then production of fluxes of reactants discussed in the last paragraph, is now part of the literature on separable loading and triggering. In general terms, we are considering the production of adequate concentrations of reactants as being loading, and then the simultaneous or later production of adequate fluxes of reactants as being triggering. It will be interesting scientifically and important practically if it is necessary to have separate loading and triggering, or if both can be done on a continuous basis in LENR generators.

Another conclusion from many experiments points to the potential importance of fluxes in causing LENR. Swartz found from his research and the literature that the production of heat, tritium and helium in LENR electrochemical experiments depended on the input electrical power.[31] He showed that there are ranges of the input power that favored the occurrence of LENR, which he called manifolds. The peaks within the ranges were termed Optimal Operating Points (OOP). This behavior could be interpreted as there being too few interactions at low input powers (and maybe also insufficient concentrations). Swartz interpreted the decrease in LENR production at input powers higher than the manifold range as being due to wasting power on extraneous electrolysis.

The above discussion of criterion for the occurrence of LENR has implicitly focused on LENR happening in the bulk (interior) of materials. However, we already noted that there is considerable evidence for LENR happening on the surface of materials. It is possible, even likely, that criteria for LENR reactions on the exterior of materials will differ quantitatively from those for LENR reactions in the interior of materials. This uncertainty does not remove the possible value of having criteria for the occurrence of LENR within materials.

We have been focused on deuteron fluxes in this section. However, it is known that in some LENR experiments there is no clear flux of the deuterium reactants. Storms measured LENR in hot gas loading experiments after the atmosphere and the palladium with dissolved deuterons have equilibrated.[32] Then, there is no concentration gradient to drive a net diffusive flux. Atomic motions and diffusion still occur, but the diffusion is equal in both directions. The consumption of deuterons inside of the material by LENR might lead to a small gradient and some remaining flux to the interior of the material. However, the steep early concentration gradients will be gone. So, the question is what supplies the dynamics needed for occurrence of LENR in this case?

There is another possible way in which deuterons can interact, even without the existence of a net flow of deuterons on or through a lattice, or small equal counter flows at equilibrium. Maybe lattice vibrations (phonons) can induce deuteron-deuteron interactions sufficient to cause LENR. There are two reasons for believing that vibrations and phonons might be the immediate cause of LENR. There are significant experimental results which show that high temperatures favor production of LENR. High temperatures increase lattice vibrations. However, the high temperatures might only enable the movement of deuterons to sites favorable for LENR or the motion of products away from reaction sites. Further, as already noted, there is recent theoretical work on the role of phonons in causing LENR.[28] A more thorough discussion of the relative importance of fluxes and vibrations in causing LENR is being written.[14]

7. Discussion

It needs to be emphasized that there is a major difference between the Lawson Criterion for hot fusion and the above criteria for LENR. The Lawson Criterion deals with the achievement of power *breakeven* in a hot fusion experiment. The LENR criteria discussed above deal with the *occurrence* of LENR, which can happen at rates far below breakeven. It remains to be seen if it is possible to develop criteria for LENR that are truly analogous to the Lawson Criteria for hot fusion. Such a development would have to confront the question of when the net power from LENR balances the input power needed to maintain the production of LENR. It seems as if that might be most possible for either hot gas or plasma loading approaches to LENR. However, a LENR criterion like the Lawson Criterion would have to be based on a power balance, independent of the means of (a) producing LENR and (b) determining whether or not LENR occurred.

The occurrence of LENR, yes or no, in a given experiment depends on the sensitivity of the means to detect the energy or matter products from LENR. Calorimetric detection of power from LENR is a useful, real-time experimental method, but it is not very sensitive. The minimum power detection level of good calorimeters is about 1 mW, which corresponds to 2.6 x 10^8 LENR per second, assuming 24 MeV per reaction. More sensitive means of detecting LENR exist and are being developed. The use of Raman scattering by Swartz might make it possible to detect LENR at levels below those needed for calorimetry.[33] Determination of helium in the gases from LENR experiments can be done in real-time and at very low levels.[34] The relative sensitivity of such methods and calorimetry remains to be determined. Detection of the transmutation products of LENR in materials can be done at low levels, even parts per billion, but that requires costly pre- and post-experiment analyses. It is not a real-time method. The amounts of transmutation products depend on the integral of the rates of various LENR over the duration of an experiment. Overall, the detection of LENR, and hence the thresholds for the occurrence of LENR shown in Figure 4, depend on the instruments and methods employed. It is expected that future work will refine the criteria discussed in the previous section.

Another important consideration relevant to determining the thresholds for the occurrence of LENR needs detailed attention. It is the well-established experimental fact that additional variations in ambient conditions and applied excitations influence the production of LENR in electrochemical experiments. LENR rates have been shown to increase with (a) temperature, (b) the input of visible, radio-frequency and terahertz electromagnetic radiation, (c) the use of ultrasound, and (d) the application of electrical impulses and magnetic fields.[35] It is one thing to increase the rate of LENR by use of such stimuli on top of adequate concentrations and fluxes, and another thing to lower the thresholds in either or both the concentration or flux of deuterons for the measurable occurrence of LENR. These issues will also require significant future research.

One more basic question concerning the plot shown in the last section is whether or not it is universal for LENR. That is, does the plot apply equally to different LENR involving different reactants in different materials, or does it need modification for various combinations of reactants and materials? The plot in the last section was mainly developed for the reactions of deuterons on or in palladium in electrochemical LENR experiments. Is it also valid for reactions involving protons, where the involved material is nickel? And, do the criteria developed above also apply to LENR experiments in which loading of protons or deuterons onto and into materials is done by using hot gases or plasmas? Again, only detailed future research can determine the answer to the question about the universality of the criteria for the occurrence of LENR that were derived and discussed above.

The issue brought up at the end of the last section also needs more attention. It deals with the roles and relative importance of reactant fluxes and lattice vibrations in the production of LENR. It seems likely that fluxes and vibrations are interrelated, since both are temperature dependent, and fluxes of ions moving in a lattice undergo collisions that might generate vibrations. Hence, a coupled approach to the question of the relative importance of fluxes and vibrations causing LENR seems necessary.

Figure 5 is an overall graphical summary of the viewpoint of this paper. It is meant to emphasize the two conditions needed for the occurrence of LENR. It also indicates the two ways in which high reactant concentrations can be reached, namely diffusion and electromigration. Also shown are three means of producing fluxes of reactions, specifically concentration gradients, electrical pulses and electromigration. The items listed in the lower right-hand part of Figure 5 are meant to indicate that lattice vibrations can be increased thermally, by the input of sound or ultrasound, or by the absorption of electromagnetic radiations. The question mark notes current uncertainty about the generation of lattice vibrations by a flux of light reactants, such as deuterons, moving through a lattice.

The availability of criteria for the occurrence of LENR might contribute to both the scientific understanding and practical commercialization of LENR. Knowing what has to be achieved could help progress toward the reproducibility and controllability of LENR energy production. Both of those features are prerequisite to the commercialization of LENR.

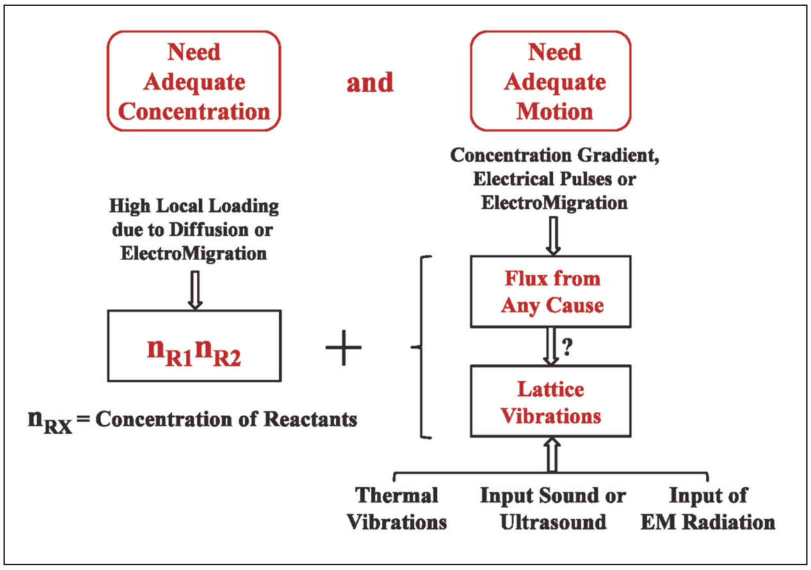

Figure 5. Graphical representation of the dual conditions that must co-exist in space and time for the occurrence of LENR, including some of the means to vary them.

Before ending, we note that the data shown in Figures 2 and 3, which formed the quantitative basis for the thresholds on reactant concentrations and fluxes in Figure 4, date from the early 1990s. Additional papers since then have strengthened the views that such concentrations and fluxes or vibrations are critical to the occurrence of LENR. However, the viewpoint offered in this paper could have been developed well over a quarter of a century ago.

Acknowledgements

The author benefited from much-appreciated email exchanges with Peter Kálmán, Steven Katinsky, Edmund Storms and Mitchell Swartz.

References

1. Nagel, D.J. 2011. “Hot and Cold Fusion for Energy Generation,” *J. Condensed Matter Nuclear Science*, 4, 1-16.
2. https://en.wikipedia.org/wiki/Ivy_Mike
3. Lawson, J.D. 1955. “Some Criteria for a Useful Thermonuclear Reactor,” A.E.R.E. report GP/R 1807, December.
4. Lawson, J.D. 1955. “Some Criteria for a Power Producing Thermonuclear Reactor,” *Proc. of the Physical Society*, Section B, 70, 6-10, doi:10.1088/0370-1301/70/1/303
5. https://www.euro-fusion.org/index.php?id=126&L=368%27A%3D0&tx_news_pi1%5Bnews%5D=20&cHash=e01416b50ff00f3d65ec92457d478ce5
6. See, for example: Chen, F.J. 2010. *Introduction to Plasma Physics and Controlled Fusion*, Springer.
7. Dooley, P. 2013. “Lawson’s Magic Formula,” *ITER Newsline*, 261, https://www.iter.org/newsline/261/1527
8. https://en.wikipedia.org/wiki/Lawson_criterion
9. https://www.fusionenergybase.com/article/measuring-progress-in-fusion-energy-the-triple-products
10. https://iopscience.iop.org/article/10.1088/0029-5515/43/9/318
11. https://lasers.llnl.gov/about/what-is-nif and https://lasers.llnl.gov/science/icf/how-icf-works
12. https://www.sandia.gov/Pulsed-Power/res-areas/inertial_confinement/
13. Fleischmann, M. and Pons, S. 1993. “Calorimetry of the Pd-D_2O System: From Simplicity via Complications to Simplicity,” *Frontiers of Cold Fusion*, Proc. of the Third International Conference on Cold Fusion, H. Ikegama, ed., Universal Academy Press, 47-66.
14. Nagel, D.J. 2021. “Where and When LENR Occur,” presented at ICCF-23.
15. Nagel, D.J. 2018. “Expectations of LENR Theories,” *J. Condensed Matter Nuclear Science*, 26, 15-31.
16. Kálmán, P. and Keszthelyi, T. 2017. “Nuclear Processes in Solids: Basic 2nd-order Processes,” arXiv:1303.1078v3, January; “Resonance-like Nuclear Processes in Solids: 3rd and 4th Order Processes,” arXiv:1303.1262v2 (January 2017); “Recoil Assisted Low Energy Nuclear Reactions,” *J. Condensed Matter Nuclear Science*, 25, 142-158 (2017); “Forbidden Nuclear Reactions,” *Physical Review C*, 99, 054620 (2019); “On Low-energy Nuclear Reactions,” arXiv:1907.05211v2 (August 2019).
17. McKubre, M.C.H. *et al.* 1992. “Excess Power Observations in Electrochemical Studies of the D/Pd System: The Influence of Loading,” *Frontiers of Cold Fusion*, Proc. of the Third International Conference on Cold Fusion, H. Ikegama, ed., Universal Academy Press, 5-18.
18. Kunimatsu, K. *et al.* 1992. “Deuterium Loading Ratio and Excess Heat Generation during Electrolysis of Heavy Water in a Palladium Cathode in a Closed Cell Using a Partially Immersed Fuel Cell Cathode,” *Frontiers of Cold Fusion*, Proc. of the Third International Conference on Cold Fusion, H. Ikegama, ed.,Universal Academy Press, 31-45.
19. https://en.wikipedia.org/wiki/Reaction_rate
20. See, for example: Mizuno, T. et al. 1992. “Cold Fusion Reaction Products and Behavior of Deuterium Absorption in Pd Electrode,” *Frontiers of Cold Fusion*, Proc. of the Third International Conference on Cold Fusion, H. Ikegama, ed., Universal Academy Press, 373-376.
21. McKubre, M.C.H. and Tanzella, F. 2012. “What is Needed in LENR/FPE Studies,” *J. Condensed Matter Nuclear Science*, 8, 187-197.
22. Storms, E. 2021. “The Nature of the D+D Fusion Reaction in Palladium and Nickel,” presented at ICCF-23.
23. McKubre, M.C.H. *et al.* 2003. “The Need for Triggering in Cold Fusion Reactions,” *Proc. of the Tenth International Conference on Cold Fusion*.
24. Fleischmann, M. and Pons, S. 1993. “Calorimetry of the Palladium-D-D_2O System,” *Proc. of the EPRI-NSF Workshop on Anomalous Effects in Deuterided Metals*, October 16-18, 1989, pp. 3-1 to 3-50.
25. Storms, E. 2016. “How Basic Behavior of LENR Can Guide a Search for an Explanation,” *J. Condensed Matter Nuclear Science*, 20, 100-138.
26. Dardik, I. *et al.* 2006. “Excess Heat in Electrolysis Experiments at Energetics Technologies,” *Proc. of the 11th International Conference on Condensed Matter Nuclear Science*, J.-P. Biberian, ed., World Scientific, 84-101.
27. Dominguez, D. *et al.* 2014. “Evidence for Excess Energy in Fleischmann-Pons-Type Electrochemical Experiments,” *J. Condensed Matter Nuclear Science,* 14, 15-28.
28. Kálmán, P. and Keszthelyi, T. 2021. “Quantum Mechanics of Nuclear Reactions in PdD_ξ Systems,” submitted to *Physical Review C*.
29. McKubre, M.C.H. *et al.* 2003. “The Need for Triggering in Cold Fusion Reactions,” *Proc. of the Tenth International Conference on Cold Fusion*.
30. McKubre, M.C.H. *et al.* 2014. “Calorimetric Studies of the Destructive Stimulation of Palladium and Nickel Fine Wires,” *J. Condensed Matter Nuclear Science,* 13, 337-345.
31. Swartz, M.R. 2008. “Optimal Operating Point Manifolds in Active, Loaded Palladium Linked to Three Distinct Physical Regions,” *Proc. of ICCF14*, p. 639.
32. E. Storms, private communication.
33. Swartz, M.R. 2018. “Increase of an Anti-Stokes Peak at the Cathode of an Electrically Driven, Active Aqueous Nickel/H_2O/Pt System,” *J. Condensed Matter Nuclear Science*, 27, 22-29.
34. See, for example: Yamaguchi, E. and Nishioka, T. 1993. “Direct Evidence for Nuclear Fusion Reactions in Deuterated Palladium,” pp. 179-188 and Bockris, J.O’M. “Tritium and Helium Production in Palladium Electrodes and the Fugacity of Deuterium Therein,” pp. 231-240, both in *Frontiers of Cold Fusion*, Proc. Third International Conference on Cold Fusion, H. Ikegama, ed., Universal Academy Press; McKubre, M.C.H. *et al.* 2000. “The Emergence of a Coherent Explanation for Anomalies Observed in D/Pd and H/Pd System: Evidence for ^{4}He and ^{3}He Production,” *Proc. of ICCF-8*.
35. Nagel, D.J. 2021. “Input Stimuli and Output Signals for LENR Experiments,” presented at ICCF-23.

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